Similitude in Stirred-Tank Reactors: Laminar Feed

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Similarity parameters are derived to correlate the selectivity of mixing controlled parallel reactions within stirred-tank reactors. For the case of laminar reactant feed, the Kolmogoroff time is shown to correlate selectivity in turbulent, semibatch reactors, provided the reactor Reynolds number $Re < Re_C$. The upper limit Re_C for the reactor Reynolds number is shown to be constrained by the volumetric flow ratio of reactant feed-to-pumping capacity. Moreover, for larger reactor Reynolds number $Re > Re_C$, the inertial time scale defined in terms of a reactor feed length is shown to correlate selectivity. Scaling laws are demonstrated from available tank data taken at up to three reactant feed points in both aqueous and viscous solutions, and for a range of vessel volumes, feed tube diameters, and feed times.

Introduction

Batch operations currently dominate bulk chemical production such as specialty organic chemicals (Paul, 1988). A common configuration is a semibatch, stirred-tank reactor with one or more laminar feedstreams (Baldyga and Bourne, 1992; Verschuren et al., 2000). Although the level of turbulence in the latter is independently controlled by the stirring rate, control of the selectivity with mixing-sensitive reactions can be difficult because of the relative importance of a number of turbulent length and time scales and their relationship to both the flow rate (or size) of the reactant feedstreams and the kinetic reaction times.

When the impeller Reynolds number in a stirred-tank reactor is sufficiently large, macroscopic properties such as power number, velocity distribution, and blending time are essentially independent of *Re*. This is also true for some smaller-scale turbulent phenomena such as mixing on a time scale proportional to the eddy dissipation or breakup time $t_d \propto N^{-1}$ where the latter represents the ratio of the turbulent integral scale $\propto L$ to the mean flow velocity $\propto NL$ (Davies, 1972). Here, N is the number of revolutions of the stirrer per unit time and L is the diameter of the stirrer blades. Mixing, however, over length scales on the order of the Kolmogoroff length ℓ_k does depend on the reactor Reynolds number, and the effect of *Re* is important in the determination of the selectivity of fast multiple reactions.

For the common case of laminar feed tubes the choice of mixing times is either the viscous (Kolmogoroff) time scale or an inertial scale (proportional to the eddy dissipation time) for small or large feed rates, respectively. This concept was useful in attempts to correlate the product distribution in stirred tanks (Baldyga et al., 1995) and other designs, such as plug-flow reactors (Forney and Chang-Mateu, 1998; Forney et al., 2001). Comparison of the inertial and viscous time scales, however, requires the computation of each mixing time to determine the maximum or rate-limiting step. Unfortunately, accurate calculation of each time scale is difficult, and both require an estimate of a universal constant whose magnitude may be disputed in the literature (Baldyga et al., 1997). Moreover, the relationship between the inertial and viscous time scales and the important operating parameters such as tank size, Reynolds number, and reactant feed rate may not be obvious.

In the present article, which considers laminar feed rates, a simple physical interpretation is provided for the choice of time scale. In fact, the form of the time scale is shown to depend only on the magnitude of a critical reactor Reynolds number, as done in a previous study of plug-flow reactors by Forney et al. (2001).

Similarity

Mixing and chemical reaction in fully turbulent liquids occurs within laminar shear layers of thickness ℓ_k, the Kolmogoroff length, between contacting eddies (Forney and Nafia, 2000). The selectivity of mixing sensitive parallel reactions in coaxial fully developed, turbulent tube and reactant feed flows from the work of Bolzern et al. (1985) was plotted as a universal curve of selectivity vs. Kolmogoroff time (Forney et al., 2001). The latter data demonstrate that the reaction rates are independent of Schmidt number Sc (or molecular diffusion times) as shown by Li and Toor (1986). In contrast, the controlling physics of mixing when the reactant feed is laminar may be limited by an inertial mixing time that depends on the feed flow rate.

Turbulence

Isotropic turbulence, although hypothetical, is a useful concept in practical flows, since many conditions more or less approach the physics of isotropy (Hinze, 1987). Experimental evidence supports the conclusion that the fine scale structure of most nonisotropic but large Reynolds number, turbulent flows is almost isotropic (local isotropy). Thus, many practical problems in turbulent phenomena that are strongly influenced by the fine scale structure can be solved to some approximation by applying the features of isotropic turbulence.

For example, the large eddies formed by the fluid motion of a mechanical impeller are not isotropic, but as the eddies decay, transferring energy to smaller fluid fragments, the smaller eddies formed become independent of the impeller geometry (Davies, 1972). In the latter case, the eddy spectrum in the equilibrium range depends only on the local eddy dissipation rate and the kinematic viscosity. Further discussion of local isotropy is provided by Brodkey (1995, p. 304) and Kresta (1998).

In the present section, the reactor Reynolds number is assumed to be sufficiently large such that the fine scale structure of the turbulence is locally isotropic. The turbulent kinetic energy κ thus can be written in the form (Hinze, 1987)

$$\kappa \propto (u')^2, \tag{1}$$

and the dissipation rate (power input per unit mass) is

$$\epsilon \propto (u')^3/\ell_e, \tag{2}$$

where ℓ_e is the average size of the energy containing eddies and u' is the root-mean-square turbulent velocity fluctuation. It is now convenient to write the eddy dissipation time ($\propto \kappa/\epsilon$) in terms of local turbulent properties or

$$t_d \propto \ell_e/u' \propto \left(\ell_e^2/\epsilon\right)^{1/3} \tag{3}$$

and the Kolmogoroff time

$$t_k \propto (\nu/\epsilon)^{1/2} \tag{4}$$

Thus, the ratio

$$\frac{t_d}{t_k} \propto \sqrt{Re_\ell} \tag{5}$$

where $Re_\ell = u'\ell_e/\nu$ is the local eddy Reynolds number (Forney and Nafia, 1998).

In the present article it is assumed that the large eddies generated by the impeller decay along the streamlines that emanate from the impeller tip. In this case, features of the turbulence are assumed to be locally isotropic and self-preserving. For example, from dimensional arguments $u' \propto \nu/\ell_e$, and it follows that the local eddy Reynolds number

$$Re_\ell = \text{constant} \tag{6}$$

along a streamline. Here, the magnitude of Re_ℓ depends on the flow boundary conditions such as impeller speed and size, in addition to the fluid kinematic viscosity.

Micromixing time scale

When the reactant feed is laminar and the feed time is large (small flow rate), experimental evidence demonstrates that the reaction rates are limited by the micromixing or Kolmogoroff time scale (Gholap et al., 1994). Experimental data suggest that the product $u'\ell_e \propto NL^2$ as the eddies decay along a streamline from the injection point (Davies, 1972), or from Eq. 6 one obtains a constant ratio

$$\frac{t_d}{t_k} \propto \sqrt{Re} \tag{7}$$

To properly correlate selectivity data within stirred tanks, it is necessary to compare the Kolmogoroff time at different tank locations for fixed fluid properties and tank geometries. Since $t_k \propto \epsilon^{-1/2}$ from Eq. 4, it is convenient to write the Kolmogoroff time at any location along a streamline in reference to its value at the maximum dissipation rate ϵ_1 near the impeller tip. Substituting $\epsilon_i = \epsilon_1(\epsilon_i/\epsilon_1)$

$$t_{ki} = t_{k1}(\epsilon_1/\epsilon_i)^{1/2} \tag{8}$$

where $\epsilon_1 \propto N^3L^2$, the Kolmogoroff time (sometimes called the viscous or micromixing time) becomes

$$t_{ki} = 1/(N\sqrt{Re})(\epsilon_1/\epsilon_i)^{1/2} \tag{9}$$

where $Re = NL^2/\nu$ is the tank Reynolds number.

The ratio of Kolmogoroff-to-flow time t_k/t_d from Eq. 7 can be interpreted as the ratio of the reaction zone length-to-reactor size since

$$u_x t_k/u_x t_d \propto 1/\sqrt{Re} \ll 1, \tag{10}$$

and is independent of streamline position where u_x is the fluid velocity along a streamline at the feed point. Similar arguments for the fractional change in the turbulent dissipation rate ϵ over the reaction zone length provide

$$1/\epsilon(d\epsilon/dx)\Delta x_k \propto 1/\sqrt{Re}, \tag{11}$$

which is also independent of a streamline position where $\Delta x_k = u_k t_k$.

From the preceding discussion, one concludes that the selectivity is at most a weak function of the local eddy dissipa-

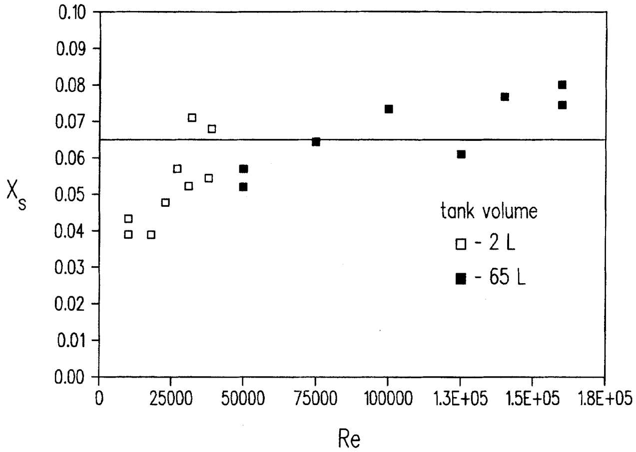

Figure 1. Selectivity vs. tank Reynolds number.

Feed position is fixed above impeller approximately half the distance to surface. $Da = 0.02$ is constant based on average ϵ_{av}. [Data are taken from Baldyga and Bourne (1988).]

tion rate with changes in feed point along the streamline, provided the Kolmogoroff time, as defined by Eq. 9, is the rate-limiting time scale. Regarding scale-up to larger tanks, Baldyga and Bourne (1988) derived an expression similar to Eq. 10 based on dimensional arguments for a fixed feed point in a stirred tank. The data in the latter work indicate a constant selectivity provided $Re > 20{,}000$ for both large and small tanks, as shown in Figure 1. For additional information, see the work of Bourne and Yu (1994), who demonstrate similar selectivity profiles in stirred tanks for similar values of the Damkohler number when the tank volume was increased by a factor of 30, with the feed position fixed near the impeller discharge. Some scatter in their experimental data was evident, however, for a feed position in the suction region of the impeller. In the latter case, the principle of local isotropy along a streamline is likely to be disturbed by the action of the impeller.

Mesomixing time scale

When the reactant feed is laminar and the feed time is small (large flow rate), experimental evidence demonstrates that the reaction rates are limited by a somewhat larger inertial or mesomixing time scale (Baldyga et al., 1995). Thus, we focus on the small feed tube in Figure 2 that is assumed to be laminar in the present study or the Reynolds number $Re_f <$ 2,500 (Davies, 1972), where

$$Re_f = 4Q_f/(\pi n d_f \nu) \tag{12}$$

Here, n is the number of feed tubes and d_f is the diameter of each feed tube. Thus, it is possible to define a reactor feed scale

$$\ell \propto (Q_f/nu)^{1/2} \tag{13}$$

as done earlier (Baldyga and Bourne, 1992; Forney et al., 2001) from conservation of mass at the feed nozzle, where ℓ represents the diameter of a thread of fluid traveling at the ambient velocity, u, of the reactor stream.

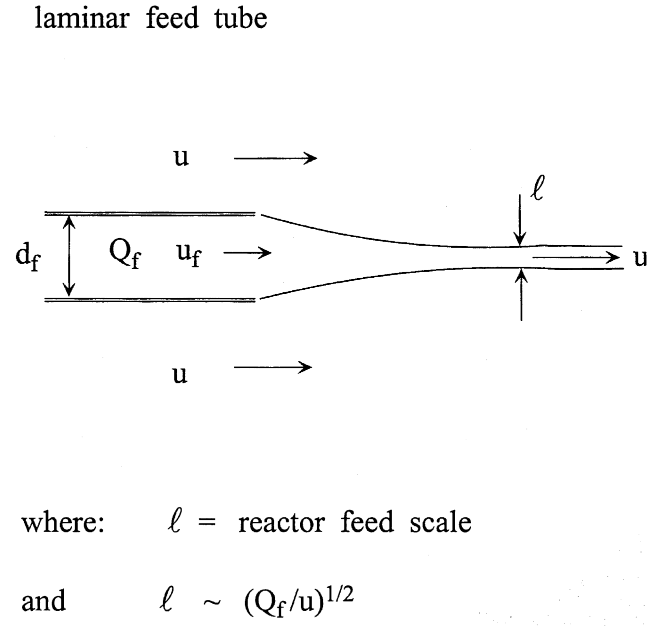

Figure 2. A laminar feed tube and length scale.

The inertial time scale t_m can now be defined from Eq. 3 by substituting ℓ for ℓ_e, as suggested by Corrsin (1964). To properly correlate selectivity data within stirred tanks, it is necessary to compare the inertial time at different tank locations for fixed fluid properties and tank geometries. Since $t_m \propto \epsilon^{-1/3}$, it is convenient to write the inertial time scale at any location along a streamline in reference to its value at the maximum dissipation rate $\epsilon_1 \propto N^3L^2$ near the impeller tip or

$$t_{mi} = t_{m1}(\epsilon_1/\epsilon_i)^{1/3}, \tag{14}$$

where $t_{m1}^3 \propto Q_f/(nN^4L^3)$. If we now define the pumping capacity of the tank in the standard form, $Q_p = 0.7NL^3$ (Verschuren et al., 2000), the inertial time scale from Eq. 14 becomes

$$t_{mi} = \left[Q_f/(nQ_p)\right]^{1/3}(1/N)(\epsilon_1/\epsilon_i)^{1/3}. \tag{15}$$

The ratio of inertial-to-flow time, t_m/t_d, can be interpreted as the ratio of the reaction zone length-to-reactor size, since

$$u_x t_m/u_x t_d \propto \left[Q_f/(nQ_p)\right]^{1/3} \ll 1, \tag{16}$$

and is independent of streamline position where u_x is the fluid velocity along a streamline at the feed point. Thus, changes in the magnitude of the inertial time scale near the feed point defined by Eq. 15 are a weak function of the eddy dissipation rate along the streamline and are likely to be of secondary importance in the determination of the selectivity.

Influence of Reynolds number

For a laminar reactant feed, the selectivity of a parallel reaction scheme is correlated with the largest of either the viscous or inertial time scale as defined earlier. The criterion for the choice of either scale is described below in terms of the reactor Reynolds number. One assumes that when the reactor feed length, ℓ, is smaller than the Kolmogoroff length, where the latter is defined by

$$\ell_k = (\nu^3/\epsilon)^{1/4} \tag{17}$$

or $\ell < \ell_k$, then the appropriate time scale for mixing is the viscous or Kolmogoroff time. In this case, the reactor feed stream is entrained between ambient turbulent eddies. However, if $\ell > \ell_k$, we assume that the time scale for mixing is the inertial scale. The choice of the mixing time is therefore determined by the magnitude of the ratio ℓ/ℓ_k, as described by Forney and Chang-Mateu (1998) and Forney et al. (2001). It should be noted that the latter concept is consistent with the ordering of the time scales, since $t_m/t_k \propto (\ell/\ell_k)^{2/3}$.

Substituting from Eqs. 13 and 17, one obtains a ratio of the feed-to-Kolmogoroff lengths of the form

$$\ell/\ell_k = \left[Q_f/(nQ_p)\right]^{1/2}(NL^2/\nu)^{3/4}(\epsilon_i/\epsilon_1)^{1/4} \tag{18}$$

Since the reactor Reynolds number $Re \propto NL^2/\nu$, setting $\ell/\ell_k = 1$ in Eq. 18 provides a transition reactor Reynolds number that is constrained by the flow ratio of reactant-to-pumping capacity in the form

$$Re_c = c(nQ_p/Q_f)^{2/3}(\epsilon_1/\epsilon_i)^{1/3} \tag{19}$$

where the universal constant c must be established from experimental data. The choice of time scale is therefore determined by the magnitude of the reactor Reynolds number. For $Re < Re_c$ the scale is the viscous or Kolmogoroff time defined by Eq. 9, while for $Re > Re_c$, the mixing time is the inertial value defined in Eq. 15.

A comparison of both scales is demonstrated in Figure 3 for a tank reactor. The values of the flow ratio of pumping capacity-to-reactant feed indicate that an increase in impeller speed shifts the mixing scale from the Kolmogoroff (micromixing) to the inertial (mesomixing) value at the transition Reynolds number $Re_c = 45{,}000$. One wishes to remain, therefore, on either the left or right of the transition value Re_c for accurate scale-up, since the physics of mixing changes at Re_c.

Micromixing ($\ell/\ell_k \le 1$)

Similitude is assured with the micromixing time scale if the tank $Re < Re_c$, the critical value defined by Eq. 19. Equal selectivity and scale-up are therefore possible for equal kinetics and stoichiometry with constant Damkohler number $Da \propto kA_o t_k$ for the feed tubes in Figure 2, where k is a rate constant. Here, A_o is defined as moles of added reactant to total volume (initial + added) such that $A_o = A_f/(1 + a)$, where A_f is the concentration of the reactant in the feed stream and a is the ratio of initial-to-added volume. Substituting for the tank Reynolds number in Eq. 9, one obtains

$$Da = (kA_o \nu^{1/2}/N^{3/2}L)(\epsilon_1 \epsilon_i)^{1/2} \tag{20}$$

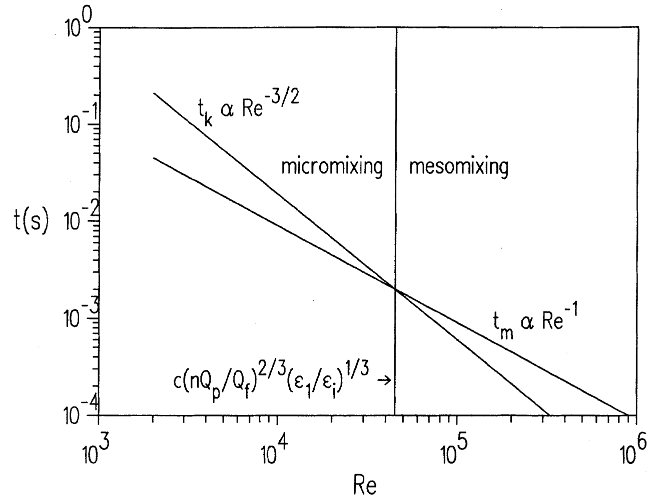

Figure 3. Mixing time vs. tube Reynolds number in stirred-tank reactor: $Re_c = 45{,}000$.

where i refers to an arbitrary feed location. In the present study, $i = 1, 2, 3$ are the feed positions near the impeller suction, fluid surface, and discharge point, respectively, as shown in Figure 4. These three feed locations are assumed to lie along a region of maximum circulation or jet pattern originating at the impeller tip.

Mesomixing ($\ell/\ell_k > 1$)

Similitude is assured with the inertial time scale if the tank $Re > Re_c$, the critical value defined by Eq. 19. Equal selectivity and scale-up are therefore possible for equal kinetics and stoichiometry with constant Damkohler number $Da \propto kA_o t_k$ for the feed tubes, where A_o is the rate-limiting reactant concentration and k is a rate constant. Substituting for the tank Reynolds number in Eq. 9, one obtains

$$Da = kA_o\left[Q_f/(nQ_p)\right]^{1/3}(1/N)(\epsilon_1/\epsilon_i)^{1/3} \tag{21}$$

where i refers to an arbitrary feed location. In the present study $i = 1, 2, 3$ are the feed positions near the impeller suction, fluid surface, and discharge point, respectively.

Correlation of Data

Micromixing

The stirred-tank data correlated in this section was taken from the work of Bourne et al. (1995) for a medium-size tank of volume $V_o = 19$ L, and Bourne and Yu (1994) for both a small tank of volume $V_o = 2.3$ L and a large tank of volume 70 L. In these experiments sodium hydroxide (A) was slowly added through a small feed tube of diameter 2 mm to a mixture of hydrochloric acid (B) and ethyl monochloroacetate (C) in the two competitive reactions below, where $k_1 \gg k_2$

$$\begin{aligned} A + B &\rightarrow P \quad (k_1) \\ A + C &\rightarrow Q \quad (k_2). \end{aligned} \tag{22}$$

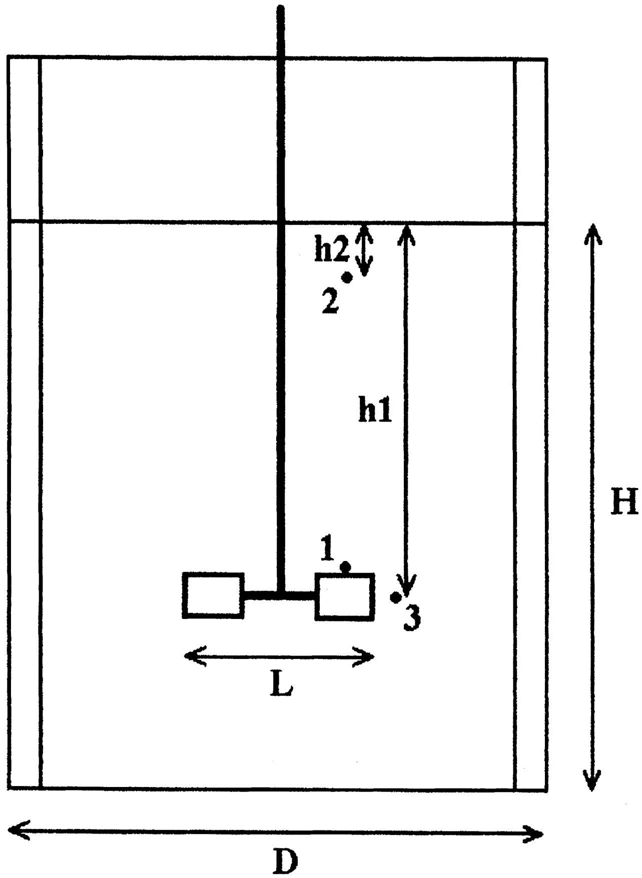

Figure 4. A 19-L stirred-tank reactor.

$D = H = 0.29$ m, $L/D = 1/3$, $r_3/D = 0.27$, $h_1 = 0.27$ m, $h_2 = 0.03$ m. Feed positions: 1—impeller suction; 2—surface; 3—impeller discharge.

The product distribution or selectivity X_Q is defined by the yield of Q relative to the limiting reagent A, that is

$$X_Q = Q/(Q + P) \tag{23}$$

The selectivity X_Q was measured at three feed locations, as shown in Figure 4, in a flat-bottomed stirred tank with four standard baffles and a six-bladed Rushton turbine whose diameter and off-bottom clearance were one-third of the tank diameter D (Gholap et al., 1994). Rotational speeds N varied from 0.6 Hz to 5 Hz, giving a range of tank Reynolds numbers of $3{,}000 < Re < 60{,}000$ that ensured fully developed turbulence.

Reactions were conducted by slowly adding one part by volume of concentrated A solution to 50 parts by volume of an equimolar solution of B and C. A stoichiometric ratio of one mol B and C to one mol of A was also maintained during the feed time, t_f. Moreover, two solutions were used: viscous (6.2 mPa·s), with 0.5 wt % HEC and aqueous (1.0 mPa·s).

Data at each feed point were chosen at sufficiently long feed times ($t_f > 1{,}500$ s) to ensure that X_Q was independent of t_f and in the micromixing controlled regime. As shown in Figures 5 and 6 the data for either different viscosity or tank size are correlated with the Kolmogoroff time defined by the Damkohler number from Eq. 20. As expected, the selectivity is lower in Figure 6 for the feed point near the turbine discharge compared to Figure 5, because of increased turbulence in the former case. The effect of turbulent intensity and thus feed location is accounted for by the definition of the Damkohler number.

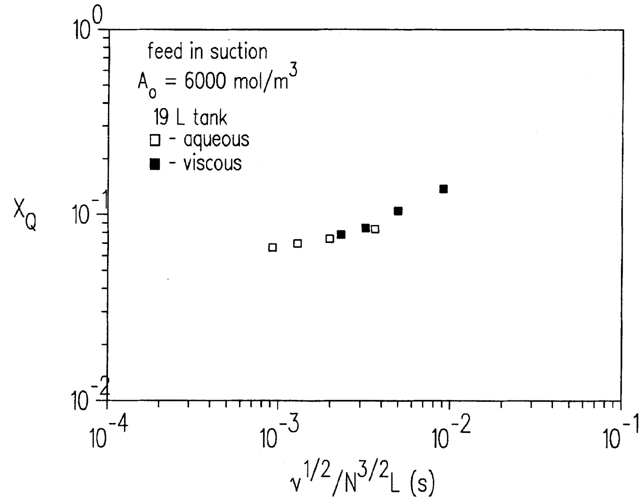

Figure 5. Selectivity vs. Kolmogoroff time in 19-L tank for feed in impeller suction. [Data from Bourne et al. (1995).]

Figure 7 plots values of the selectivity X_Q vs. the Damkohler number Da defined by $k_2 = 0.023$ m^3/mol·s. The dimensionless group appears to properly correlate the data without speculation on the size of the reaction zone. The ratio of turbulent dissipation rates ϵ_i/ϵ_1 used to account for the location of the feed points was taken from the data listed in Bourne and Yu (1994), Baldyga and Bourne (1988), and Laufhuette and Mersmann (1985). From these sources, $\epsilon_1/\epsilon_{av} = 15$ for the location of the turbine suction feed and 0.2 for

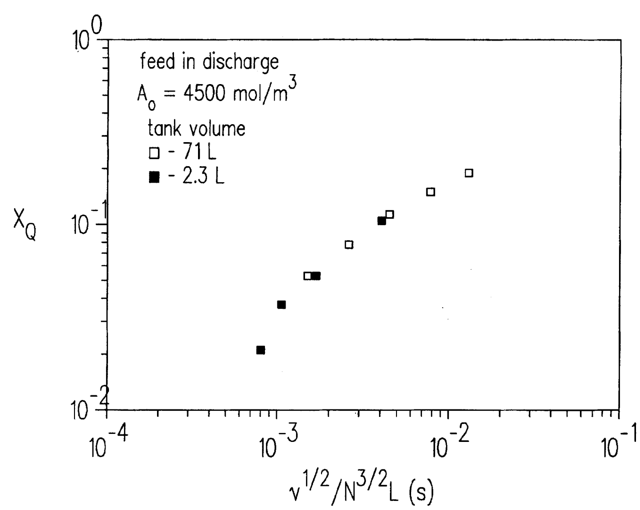

Figure 6. Selectivity vs. Kolmogoroff time for feed in impeller discharge. [Data from Bourne and Yu (1994).]

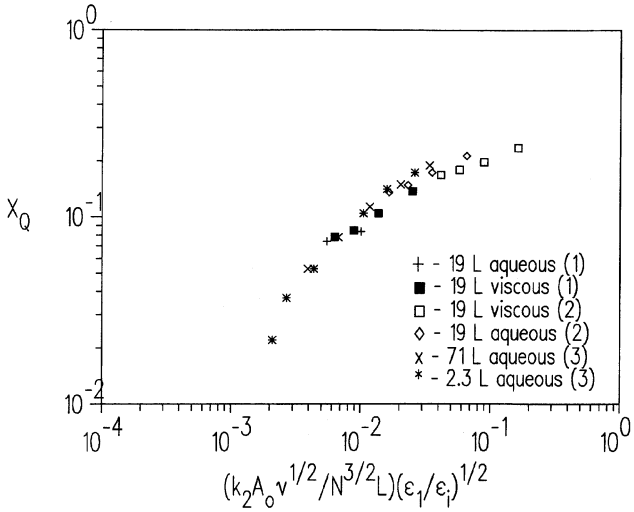

Figure 7. Selectivity vs. Damkohler number for a stirred tank with $Re < Re_c$. [Data from Bourne and Yu (1994) and Bourne et al. (1995).]

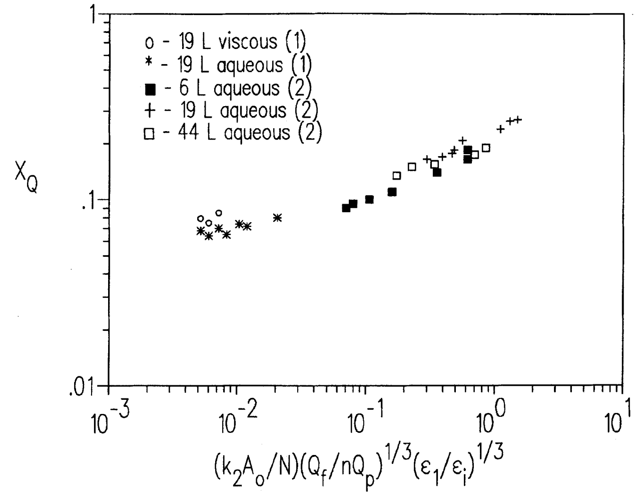

Figure 8. Selectivity vs. Damkohler number for a stirred tank defined by the inertial time scale with $Re > Re_c$. [Data in feed position 2 from Verschuren et al. (2000); data in feed position 1 from Bourne et al. (1995) and Gholap et al. (1994).]

the surface feed position, or in the latter case one assumes $\epsilon_1/\epsilon_2 = 75$. The turbulent dissipation rate ϵ_3 for the discharge feed point was estimated to be 64% of ϵ_1, since $\epsilon_3/\epsilon_1 = \exp(-9.96\Delta r/D)$, where $\Delta r/D = 0.045$ represents the shift in the discharge feed point relative to the position of maximum ϵ at $r/D = 0.23$ (Bourne and Yu, 1994).

Evaluation of the critical Reynolds number requires knowledge of the universal constant c that appears in Eq. 19. This constant can be determined by observing the feed time t_f at which the selectivity becomes horizontal on a plot of X_Q vs. t_f such as the aqueous curve in Figure 2 of Bourne et al. (1995). Although there is some variation in the estimate of c, it appears that a value of $c = 50$ is a reliable number and that was used in the present study to evaluate product distributions.

Mesomixing

The stirred-tank data correlated in this section was taken from the work of Verschuren et al. (2000) for three geometrically similar tank volumes of 6, 19, and 44 L equipped with a standard geometry of a six-bladed Rushton impeller with four baffles, as described in the previous section. These data were chosen since the diameters of the feed tubes were large at values of $d_f = 5$, 8, and 10 mm, respectively, for the indicated tank volumes. Moreover, the feed times of sodium hydroxide (A) were short $t_f < 500$ s, and the feed volume was a relatively large 1/20 of the vessel volume for the same parallel reactions of Eq. 22.

Under these conditions the ratio of reactant feed-to-pumping capacity, for example, was a relatively large $Q_f/Q_p > 1.53 \times 10^{-3}$ for the 19-L tank (compared to $Q_f/Q_p > 4.4 \times 10^{-5}$ for the data of Bourne et al., 1995). Calculation of the critical Reynolds, Re_c, at an impeller speed of $N = 4$ Hz and a feed time $t_f = 250$ s gives a value of $Re_c = 14{,}400$. The tank Reynolds number, however, was $Re = 36{,}860$, so that $Re > Re_c$ and the inertial time scale is shown to be the important mixing time.

The data from Verschuren et al. (2000) are plotted in Figure 8 vs. the Damkohler number defined by Eq. 21. Included in Figure 8 are five data points of Bourne et al. (1995) for a 19-L vessel at values of the impeller speed $3 < N < 5$ Hz, where the selectivity of both the viscous and aqueous solutions for reaction Eq. 22 were converging, suggesting independence of the viscous time scale. Also included are data from Gholap et al. (1994) for $Re > Re_c$ that represent the product distribution for a diazo coupled reaction.

Although the Damkohler number for mesomixing is expressed in terms of Q_f/Q_p, it is worth noting that substitution of the impeller diameter L and the speed N at fixed feed location gives

$$Da \propto \left(1/t_f N^4\right)^{1/3}. \qquad (24)$$

Under these circumstances the product distribution would depend only on the impeller speed N at fixed feed time t_f, as determined in a previous study by Tipnis et al. (1994) and not the power input per unit volume $\epsilon_{av} \propto N^3L^2$. Moreover, if one scales a reactor up at constant ϵ_{av}, the feed time for constant product distribution would vary as $t_f \propto L^{8/3}$, as determined by Bourne and Hiber (1990).

Conclusion

Mixing controlled parallel reactions in turbulent, stirred-tank reactors with laminar feed streams are shown to be correlated with either a viscous or an inertial time scale. The choice between time scales is shown to depend on the magnitude of a transition reactor Reynolds number Re_c, that is constrained by the volumetric flow ratio of reactant-to-pumping capacity.

For a reactor Reynolds number $Re < Re_c$, the Damkohler number defined in terms of the viscous time scale (micromixing) is sufficient to scale selectivity. In contrast, if $Re > Re_c$, the inertial time (mesomixing) is the appropriate scale. The selectivity is shown to be insensitive to the Reynolds number, provided $Re > 10{,}000$ to ensure fully developed turbulence. In general, the product distribution does not depend explicitly on either the Schmidt number Sc or Reynolds number Re. Moreover, similitude with constant energy dissipation rate (power per unit mass) was only applicable for constant viscosity fluids in the micromixing regime.

The forms of the Damkohler number that include a local eddy dissipation rate are shown to be useful to characterize reactant feed locations along major fluid streamlines emanating from the impeller tip and in the scale-up to larger tank volumes. Properties of locally isotropic, decaying turbulence along such streamlines demonstrates that the reaction length is small relative to the tank dimension and that the use of a local eddy dissipation rate at the feed discharge point is justified. That is, eddy decay in which the turbulent length and time scales change with position as the reactant is convected away from the injection point is normally of secondary importance and should not complicate scale-up.

Although the correlations provided are for a particular Rushton impeller and tank geometry, similar correlations of product distribution vs. Damkohler number would result, but be of somewhat different shapes for other tank geometries.

Notation

a = ratio of initial tank-to-added volume
A_o = reactant concentration ($= A_f/(1+a)$)
A_f = reactant feed concentration
c = universal constant ($= 50$)
d = vertical width of impeller blade, m
D = tank diameter, m
Da = Damkohler number
d_f = feed-tube diameter, m
H = liquid depth in tank, m
h_1 = impeller depth, m
h_2 = depth of surface feed, m
k = rate constant, $m^3\ mol^{-1}\ s^{-1}$
L = impeller diameter, m
ℓ = reactant feed length scale, m
ℓ_e = integral length scale, m
ℓ_k = Kolmogoroff length, m
N = impeller speed, s^{-1}
n = number of feed tubes
Q_f = volume flow rate of limiting reactant, $m^3 \cdot s^{-1}$
Q_p = stirred-tank pumping capacity = $0.7NL^3$, $m^3 \cdot s^{-1}$
Re = tank Reynolds number = NL^2/ν
Re_f = feed-tube Reynolds number
Re_ℓ = eddy Reynolds number
r_3 = radius of discharge, m
Sc = Schmidt number
t_k = Kolmogoroff time, s
t_d = eddy dissipation time, s
t_m = inertial time scale, s
t_f = reactant feed time, s
u = local fluid velocity, $m \cdot s^{-1}$
u' = root-mean-square turbulent velocity fluctuation, $m \cdot s^{-1}$
u_f = fluid velocity in feed tube, $m \cdot s^{-1}$
u_x = local fluid velocity along streamline
x = distance along a streamline, m
X_Q = selectivity or product distribution

Greek letters

ϵ = turbulent energy dissipation, $m^2 \cdot s^{-3}$
κ = turbulent kinetic energy, $m^2 \cdot s^{-2}$
ν = kinematic viscosity, $m^2 \cdot s^{-1}$
ρ = fluid density $kg \cdot m^{-3}$

Subscripts

i = feed location in stirred tank
1,2,3 = impeller suction, liquid surface, and impeller discharge, respectively

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